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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XXXV. *March* 12, 1875. No. 5.

PROFESSOR J. C. ADAMS, F.R.S., President, in the Chair.

Major-General J. C. Hannyngton, India Office, S.W.;
Prof. Robert Stawell Ball, F.R.S., M.R.I.A., University, Dublin;
H. Savile W. Evans, Esq., Wimbledon Park House;
A. M. W. Downing, Esq., B.A., Royal Observatory, Greenwich;
John Dreyer, Esq., M.A., Observatory, Parsonstown, Ireland;

were balloted for and duly elected Fellows of the Society.

On the Method to be used in Reducing the Observations of the Transit of Venus, 1874, *December* 8. By Sir G. B. Airy, K.C.B., Astronomer Royal.

In the Method of Reduction of the Observations of the late Transit of *Venus* which I now submit to the Royal Astronomical Society, there is no mathematical novelty. But the conditions of this series of observations are very different from those of any other observations known to me. Different nations and different observers are concerned in the observations; different theoretical views have animated the astronomers who have selected the stations, and planned the observations; instruments of different classes have been employed; different elements have been the subject of observation. I have, therefore, thought it well to call the attention of the Society to a form of reduction which will include in one group the observations made at all places, and by all methods; and in this form I hope the whole body of observations will be reduced, at least, in the first instance. It possesses this great value, amongst others, that it enables us at once to point out a faulty observation, which otherwise might lurk undiscovered, but producing a baneful effect on the conclusions. Those investigators, however, who desire to ascertain the inferences properly deducible from limited classes of observations, will find that this method gives them every facility. The equations to be used are ready to hand, and it is only necessary to separate them from the others. Still I hope that in the final reductions the whole mass of observations will be used in one group. Every

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individual observation will then have an influence proportional to its merit ; and reason, as well as formula, leads us to think that the best final result will thus be obtained.

1. For the first step, we must call for the assistance of the computers of Ephemerides, or rather for the assistance of one computer of Ephemeris, as it is essential that the same elements and methods of computation of tabular places be employed throughout. The following elements must be computed with the utmost precision to $0''\cdot01$; second powers of time being used where necessary. It is not injurious to the result that these elements be wrong by a second or two ; but it is of the utmost importance that the same errors be continued throughout without the slightest alteration.

2. The cosmical elements upon which the phenomena of the Transit depend are—

The RA and NPD of the Sun's centre.
The Sun's angular radius.
The equatorial horizontal parallax of the Sun.
The RA and NPD of the centre of *Venus*.
The angular radius of *Venus*.
The equatorial horizontal parallax of *Venus*.

These elements ought to be prepared for intervals throughout the entire Transit, from a short time, say a minute, before the first external contact, as visible from the Earth's surface, to an equal time after the last external contact as visible from the Earth's surface. During the middle part of the Transit, when there is no question of close micrometer-measure or Janssen-measure, it may suffice to make these calculations for every minute of *sidereal* time ; but from the internal approach of limbs, say a minute or more from internal contact to a minute from external contact (or *vice versâ*, the minute-ranges being always interpreted in extension of the intervals here treated) the calculations ought to be made for every ten seconds of time.

3. To these elements are to be attached, where necessary, symbolical variations. We may assume the RA and NPD of the Sun's centre to be correct; any real error in them being afterwards applied to the similar elements for *Venus*. If our observations consisted merely of ocular observation of contacts, we might also assume the Sun's angular radius to be correct; as an error in it might be neutralised by errors of the RA and NPD of the centre of *Venus*. But an erroneous value of the Sun's radius would produce errors of different magnitudes in the measures of photographs made at different periods of the Transit (converted into sidereal measure by proportional parts of the Sun's photographic radius corrected for photographic irradiation), and therefore we must suppose the Sun's radius liable to error. This must be connected with corresponding errors of RA and NPD of *Venus* ; and the Sun's angular radius for ocular observations (corrected for irradiation) must now be supposed liable

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to the same error. Also, viewing the difference of the modes in which the disk of *Venus* will be treated, as affecting eye-observations of contact, and as affecting micrometer-observations or photographic records, we must consider the angular radius of *Venus* in contact-observations as liable to error. The RA and NPD of *Venus*, and both the parallaxes, are evidently to be taken as possibly erroneous. Thus we must use the following expressions, in which P and p are the numerical tabular values of the eq. hor. parallax of the Sun and *Venus*; p being considered $= n.\mathrm{P}$, where n is the proportion of the distance of the Sun from the Earth to the distance of *Venus* from the Earth: R being the numerical tabular value of the angular radius of the Sun, and r that of *Venus*.

Angular radius of the Sun = $\mathrm{R} + \delta.\mathrm{R}$.

Sun's equatorial horizontal parallax = $\mathrm{P} + \delta.\mathrm{P}$.

Geocentric RA of the centre of *Venus* = Tabular ♀ RA + δ. ♀ RA.

Geocentric NPD of the centre of *Venus* = Tabular ♀ NPD + δ. ♀ NPD.

Angular radius of *Venus*, for contact-observations = $r + \delta.r$.

Equatorial horizontal parallax of *Venus* = $p + \delta.p$.

= $n.\mathrm{P} + n.\delta.\mathrm{P}$.

4. I proceed to point out the principal steps of the first computation in detail; partly for the purpose of indicating the various sources of the errors which ought to be taken into account in the computations conducted on the Theory of Probabilities; and partly to give facilities to the calculations of each computer by exhibiting in an orderly way the succession of steps of computation.

5. The transit-clock-time of each observation is to be taken, subject to error. And here it is to be observed that, where there are, at one place, multiple observations (as eye-observations both of ingress and of egress, or numerous micrometrical measures or photographic records), even though the probable errors of all the observations may be fairly considered equal, yet every individual observation must be considered as having its own independent Actual Error, up to the very last step of reduction where Probable Errors are introduced.

6. To this clock-time must be applied the correction of transit-clock, subject to error (probably so small that it may usually be neglected, but equally applicable to every one of multiple observations if it be retained). Thus the Local Sidereal Time of each separate observation is obtained.

7. The Terrestrial Longitude measured from the standard point of the Ephemeris is to be applied, subject to its possible error.

8. Thus is obtained, for each separate observation, the

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Sidereal Time at the Ephemeris-standard-point; subject to the errors of transit-clock-time of observation, of transit-clock-correction, and of terrestrial longitude, and requiring corresponding corrections: expressed in terms of 1^s of Sidereal Time.

9. By use of these times there are to be computed for every observation,—

The Geocentric RA and NPD of the Sun's centre, affected by multiples of all the corrections mentioned above.

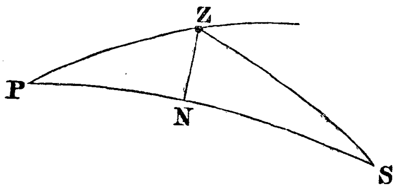
The Sun's angular radius (afterwards to be increased numerically by "augmentation," which is numerically sensible), affected by the symbolical correction δ R.

The Sun's equatorial horizontal parallax (afterwards to be corrected numerically for geocentric radius of station), affected by the symbolical correction δ P.

The Geocentric RA and NPD of the centre of *Venus*, affected by multiples of all the corrections mentioned above, and also by the symbolical corrections δ. ♀ RA and δ. ♀ NPD.

10. Combining the Geocentric RA with the Local Sidereal Time in (6), the hour-angle is found. Symbolical corrections are not required, as this element is used only in computing small quantities (parallax in RA and in NPD). The Astronomical Latitude of Station is to be corrected by applying Angle of Centre so as to obtain Geocentric Latitude. The Eq. Hor. Parallax is to be multiplied by the proper elliptic factor to produce Local Horizontal Parallax. Then the following calculation is to be made.

11. In the diagram below, P is the pole above the horizon, Z the geocentric zenith, S the object (Sun's centre or *Venus'* centre), ZN an arc perpendicular to PS. Then,



$$\text{Sine ZN} = \text{Sine PZ} . \text{Sine} \angle \text{P}.$$

$$\text{Tan PN} = \text{Tan PZ} . \text{Cosine} \angle \text{P}.$$

$$\text{NS} = \text{PS} - \text{PN}.$$

$$\text{Factor of parallax in RA} = \frac{\text{Sine ZS} \times \text{Sine} \angle \text{S}}{\text{Sine PS}} = \frac{\text{Sine ZN}}{\text{Sine PS}};$$

$$\text{or} = \frac{\text{Sine PZ} . \text{Sine} \angle \text{P}}{\text{Sine PS}}.$$

$$\text{Factor of parallax in NPD} = \text{Sine ZS} . \cos \angle \text{S} = \text{sine ZS} . \tan \text{NS} . \cot \text{ZS}$$

$$= \cos \text{ZS} . \tan \text{NS} = \cos \text{ZN} . \text{sine NS}.$$

These factors should be computed separately for the Sun and for *Venus*.

12. Applying these parallaxes to the computed Geocentric Places, introducing all the symbolical corrections, and using the

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symbol (n^r) to indicate the numerical value of the factor which in each place multiplies the symbolical corrections, we have the following Apparent Quantities:—

RA of Sun's Centre = Tabular $\odot$ RA $+ (n^r) \times \delta$. Terrestrial Longitude.
$+ (n^r) \times$ P $+ (n^r) \times \delta$. Clock Time of Observation.
$+ (n^r) \times \delta$.P.

NPD of Sun's Centre = Tabular $\odot$ NPD $+ (n^r) \times \delta$. Terrestrial Longitude.
$+ (n^r) \times$ P $+ (n^r) \times \delta$. Clock Time of Observation.
$+ (n^r) \times \delta$.P.

RA of *Venus*' Centre = Tabular ♀ RA + δ.♀ RA.
$+ (n^r) \times$ P $+ (n^r) \times \delta$. Terrestrial Longitude.
$+ (n^r) \times \delta$. Clock Time of Observation
$+ (n^r) \times \delta$.P.

NPD of *Venus*' Centre = Tabular ♀ NPD + δ.♀ NPD.
$+ (n^r) \times$ P $+ (n^r) \times \delta$. Terrestrial Longitude.
$+ (n^r) \times \delta$. Clock Time of Observation
$+ (n^r) \times \delta$.P.

where the symbol (n^r) has a different numerical value in every different place, excepting that for each element the values of (n^r) will be the same for P and δ.P, though differing from one element to another.

For the moment, we may use the following abbreviated expressions:—

Apparent RA of Sun's Centre = Tabular Apparent $\odot$ RA + $\Delta.(\odot$ RA$)$,
Apparent NPD of Sun's Centre = Tabular Apparent $\odot$ NPD + $\Delta.(\odot$ NPD$)$,
Apparent RA of *Venus*' Centre = Tabular Apparent ♀ RA + Δ.(♀ RA),
Apparent NPD of *Venus*' Centre = Tabular Apparent ♀ NPD + Δ.(♀ NPD),

where the term "Tabular Apparent" is used to denote "Tabular Geocentric Place affected by Tabular Parallax."

And these we shall further contract to

$\odot$ RA + $\Delta\,(\odot$ RA$)$,
$\odot$ NPD + $\Delta\,(\odot$ NPD$)$,
♀ RA + Δ (♀ RA),
♀ NPD + Δ (♀ NPD).

13. From these the Tabular Apparent Distance of the Centre of *Venus* from the Centre of the Sun will be computed by the following formula, of which the error is far below $0''{\cdot}01$:

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$$\sqrt{\{\sin(\odot\,\mathrm{NPD}).\sin\text{♀}(\mathrm{NPD}).(\odot\,\mathrm{RA}-\text{♀}\,\mathrm{RA})^2+(\odot\,\mathrm{NPD}-\text{♀}\,\mathrm{NPD})^2\}}$$

(which line we may call Uncorrected Tabular Distance)

$$+\frac{\sin(\odot\,\mathrm{NPD}).\sin\text{♀}(\mathrm{NPD}).(\odot\,\mathrm{RA}-\text{♀}\,\mathrm{RA})}{\text{Uncorrected Tabular Distance}}\times\Big(\Delta(\odot\,\mathrm{RA})-\Delta(\text{♀}\,\mathrm{RA})\Big)$$

$$+\frac{\odot\,\mathrm{NPD}-\text{♀}\,\mathrm{NPD}}{\text{Uncorrected Tabular Distance}}\times\Big(\Delta(\odot\,\mathrm{NPD})-\Delta(\text{♀}\,\mathrm{NPD})\Big).$$

In these terms, $\odot\mathrm{RA}-\text{♀}\,\mathrm{RA}$ and $\odot\mathrm{NPD}-\text{♀}\,\mathrm{NPD}$ are to be expressed in arc, to the accuracy $0''\cdot01$.

The most convenient way of effecting the numerical calculation will be, by use of an auxiliary angle ϕ.

$$\text{Let } \tan\phi=\frac{(\odot\,\mathrm{RA}-\text{♀}\,\mathrm{RA})\times\sqrt{\{\sin(\odot\,\mathrm{NPD})\times\sin(\text{♀}\,\mathrm{NPD}\}}}{\odot\,\mathrm{NPD}-\text{♀}\,\mathrm{NPD}},$$

then the uncorrected tabular distance of centres is given by either of the following expressions :—

$$\frac{1}{\sin\phi}\times(\odot\,\mathrm{RA}-\text{♀}\,\mathrm{RA})\times\sqrt{\{\sin(\odot\,\mathrm{NPD})\times\sin(\text{♀}\,\mathrm{NPD})\}},$$

$$\frac{1}{\cos\phi}\times\{\odot\,\mathrm{NPD}-\text{♀}\,\mathrm{NPD}\};$$

and the corrected distance of centres is

$$\begin{aligned}&\text{uncorrected tabular distance of centres}\\&+\sin\phi\times\sqrt{\{\sin(\odot\,\mathrm{NPD})\times\sin(\text{♀}\,\mathrm{NPD})\}}\times\{\Delta(\odot\,\mathrm{RA})-\Delta(\text{♀}\,\mathrm{RA})\}\\&+\cos\phi\times\{\Delta(\odot\,\mathrm{NPD})-\Delta(\text{♀}\,\mathrm{NPD})\}.\end{aligned}$$

14. The expression just found is that which will apply when the subject of measure is the distance of centres of Sun and *Venus* at an observed time, as for instance in the measures of the distance of centres in photographs. It will be compared with the proportional part of the " Sun's augmented semidiameter affected with δR ", the proportion being taken from the measures of photographs, corrected as may be necessary. But when the phenomenon observed is the internal contact, the radius of *Venus* must be applied, and we have for corrected distance of the external limb of *Venus* from the Sun's centre,

$$\begin{aligned}&\text{uncorrected tabular distance of centres}+r\\&+\sin\phi\times\sqrt{\{\sin(\odot\,\mathrm{NPD})\times\sin(\text{♀}\,\mathrm{NPD})\}}\times\{\Delta(\odot\,\mathrm{RA})-\Delta(\text{♀}\,\mathrm{RA})\}\\&+\cos\phi\times\{\Delta(\odot\,\mathrm{NPD})-\Delta(\text{♀}\,\mathrm{NPD})\}\\&+\delta r.\end{aligned}$$

This will be compared with the Sun's augmented semidiameter affected with δR.

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15. In order to form the next combination of the equations given by the separate observations, we must be prepared to assign numerical values to the Probable Errors, or to the Probable Values of the Chance-Errors (of Local Time of Observation of the Phenomenon, or of Measure of the Photograph, as the case may be). For these estimations I do not think it is possible to lay down any mathematical rule. I think that each observer must give his careful attention to the circumstances, and must on his own judgment decide upon the value of the observed or measured element which he thinks best, and upon the range + or − which he thinks may be admitted as including half the possible errors. The measure of the half range (that is, from error = 0 to error = + value, or from error = 0 to error = − value) will be the Probable Error. We will now consider the application of this Probable Error under different circumstances.

16. First. Suppose that the observation has been the simple observation of internal contact, at a place whose longitude is accurately known. At that time, from the nature of the observation, the apparent distance of centres + the angular radius of *Venus* = Sun's augmented semidiameter. But the tabular calculation of apparent distance + angular radius of *Venus* gives a value (presumably) different from the Sun's augmented semidiameter; say, less by a quantity which we will call "Discordance." To remove this, we must apply corrections to the elements ♀ RA, ♀ NPD, R, r, P, and must convert all such quantities as those expressed by Δ in articles 13 and 14 into terms of $\delta.$♀ RA, $\delta.$♀ NPD, $\delta.$R, $\delta.r$, $\delta.$P; and must introduce a Chance-Error of Observed Time; so that,

$$\text{Sun's augmented semidiameter} + \delta.\mathrm{R} =$$
$$= \left\{\begin{array}{l}\text{Value of tabular calculation of apparent distance of centres} + r \\ + (n^r) \times \delta.\text{♀RA} + (n^r) \times \delta.\text{♀NPD} + \delta.r + (n^r) \times \delta.\mathrm{P} \\ + (\mathrm{N}^r) \times \text{Chance-Error of Observed Time.}\end{array}\right\}.$$

Or, removing all but the last term to the left hand,

$$\text{Discordance} - (n^r) \times \delta.\text{♀RA} - (n^r) \times \delta.\text{♀NPD} + \delta.\mathrm{R} - \delta.r - (n^r) \times \delta.\mathrm{P} =$$
$$= (\mathrm{N}^r) \times \text{Chance-Error of Observed Time.}$$

The corrections of ♀ RA, ♀ NPD, R, r, P, are, so far as they enter into this equation, definite quantities, free from chance-error; but the Observed Time has a Chance-Error; and has, therefore, a Probable Error, a definite quantity, to be determined as has been mentioned. Divide the equation by (N^r) × the Probable Error of Observed Time; and (remarking that, when a chance-error is divided by a definite quantity, its probable error is divided by the same definite quantity) we have

$$\frac{\text{Left Side of Equation}}{(\mathrm{N}^r) \times \text{Probable Error of Observed Time}} = \frac{\text{Chance-Error of Observed Time}}{\text{Probable Error of Observed Time}},$$

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giving an equation in which the probable error of the second side = the abstract number 1. This is the state to which every observation must be reduced. The right side of the equation (which, if the observations were perfect, would = 0), is of no use, except as enabling us to say that the left side, as divided by $(N^r) \times$ Probable Error of Observed Time, is = 0 with Probable Error = ± 1.

17. Second. Suppose that, with the ocular observation of internal contact, there are to be combined micrometer-observations, Janssen photographs, or any other measures, each of which alone can be interpreted as giving both an independent determination of the time of internal contact, and an estimate of the probable error of time of such independent determination. Put *p.e.* (1), *p.e.* (2), &c., for the probable errors of the ocular determination and the other independent determinations. Then, by a well-known theorem, the independent determinations are to be combined with the weights $\left(\frac{1}{p.e.\ (1)}\right)^2$, $\left(\frac{1}{p.e.\ (2)}\right)^2$, &c., to form the Observed Time for which all the calculations from the Ephemeris are to be made. And the quantity

$$\frac{1}{\text{Probable Error of Observed Time}} = \sqrt{\left\{\left(\frac{1}{p.e.\,(1)}\right)^2 + \left(\frac{1}{p.e.\,(2)}\right)^2 + \text{\&c.,}\right\}}$$

which, with $\frac{1}{(N^r)}$, determines the factor of the left side of the equation.

18. Third. Suppose that the longitude of the station is not known with indisputable accuracy: and that observations are made similar to those of Articles 16 and 17. It will be remarked here that the correction of the longitude of the station, which is to be sought by independent means, is not a thing of the same class as the corrections of ♀ RA, P, &c., which are the special object of the observations of the Transit of *Venus*; nor is it a subject of observations of the moment, and liable to the accidental errors of the moment, like the observations of contacts, &c. Also, it may be expected that the longitude will in no long time be accurately determined. In this case then it will be proper, in preparing the terms on the left side of the equation, to attach to the clock-time of observation in Article 12 a symbolical correction $\delta.l$, and to combine its effect, in the shape of $(n^r) \times \delta.l$, with the "tabular calculation of apparent distances of centres" and therefore with the "Discordance." Then, in the final solution of equations, every correction $\delta.$ ♀ RA &c., $\delta.$P &c., will contain a term multiplying $\delta.l$. And when the numerical value of $\delta.l$ shall be subsequently determined, the requisite alteration of the numerical values of $\delta.$ ♀ RA &c., $\delta.$ P &c., can then be effected at once.

The evaluation of the "Probable Error of Observed Time" is the same as when the longitude is certainly known.

These remarks apply to the treatment of all classes of observation, at every station where the longitude is uncertain.

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19. Fourth. Suppose that observations are made at several stations, all of which depend for their longitude on one station, whose longitude is somewhat uncertain. In this case, the symbolical correction $\delta.l$ must be introduced into the calculations for each station; and then perhaps it will be simplest to exhibit separately the equations for each station. But they may be combined into one by using the combination-weights

$\frac{1}{(\text{Probable Error of Observed Time})^2}$ proper for each: and that one resulting equation will contain a multiple of $\delta.l$.

20. Fifth. Hitherto we have confined our remarks to the observation of one phenomenon (Ingress or Egress), though with variations in the methods of observing and in the localities (supposed to be not widely separated). Now, suppose that at a station whose longitude is accurately known the Ingress and Egress are both observed.

As in Article 16, each phenomenon will give one equation; and, if the planet's geocentric place were subject to no error of RA or NPD, a favorable equation for Parallax would be obtained by taking the difference of these equations. But the errors of geocentric RA and NPD are sufficient to render this inaccurate. And although a great part of this inaccuracy would be removed by combination with a similar pair of equations under different terrestrial latitude, still there would remain a part, and the effects of different errors of observation might also interfere injuriously. It will be better to retain the equations uncombined.

21. Sixth. Suppose that Ingress and Egress are both observed at a station whose longitude is uncertain.

The inference is the same as in the last case. It will be best to retain the two equations given by the two phenomena, but both must contain the same longitude-correction $\delta.l$.

22. Seventh. Suppose that, during the progress of the Transit, a series of photographs is taken, at times that are subject to no sensible error except a possible error of longitude common to all; and suppose that measures of the photographs are taken, which give the "distances" of the centre of *Venus* from the centre of the Sun, in terms of the Sun's diameter on the photograph (corrected for photographic irradiation and for the effect of refraction on the apparent length of the inclined "distance," upon which corrections, however, we enter into no details here), subject to small independent errors in the measures or interpretations of the photographs. This case is different in some points from those previously considered. The changes in the relative positions of the Sun and *Venus* are so great that it is impossible to take a mean of the equations given by different photograph-measures. The element of uncertainty is not in time, but in measures. The equations must therefore be treated thus. The symbol for the correction of longitude $\delta.l$ must be kept on the left side of the equation, and multiplied in the successive operations by the proper factors, as in Articles 18, 19, 21. The chance-error of each measure must be kept on the right side, expressed

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by the same unit (perhaps $1''\cdot00$) in which the computed measure of distances is expressed. The probable error of measure is to be expressed by the same unit. Then, on dividing by that probable error, an equation for each measure will be obtained whose probable error is = abstract 1, and which can be used in combination with all the other equations (produced by observations of a totally different kind) each of whose probable errors is = abstract 1.

23. The instances which have been given will probably suffice to guide the computer in forming the equations for any other combination of observations not described here. It remains to point out the steps of the well-known processes for ultimate treatment of the collected equations.

24. First, it is to be remarked that any theorist who is desirous of examining the validity of special combinations of stations (as, for instance, stations adapted to determination of result by difference of duration of Transit, or stations giving comparison of absolute times of the same phenomenon of Transit), or who desires to examine the results of special methods of observation (as, for instance, by photographs as distinguished from eye-observations), will find here all the materials ready to his hand. He has merely to extract the equations applying to his stations or method, and to combine them and treat them in the same manner as that in which the whole mass of equations are to be combined and treated. I earnestly hope, however, that the whole mass may be used in combination.

25. Next, as all the equations are so reduced as to be subject each to the probable error = abstract 1, the treatment is simple. Multiply every equation by its numerical coefficient of δ. RA of centre of *Venus* (omitting consideration of Probable Error), add together these multiplied equations, and one Final Equation is obtained. Multiply every equation by its numerical coefficient of δ. NPD of centre of *Venus*, add together the multiplied equations, and a second Final Equation is obtained. Similarly with regard to the numerical coefficients of δ . R, δ . Semidameter of *Venus*, δ . Horizontal Equatorial Parallax of the Sun. These five equations will give the five quantities sought. It will be remembered that every value derived from the equations will contain one term depending on each of the quantities $\delta\, l$.

26. The determination of probable error of final results, always troublesome, will perhaps be least troublesome in the following manner. In the right side of the equations which have been divided by Probable Error, instead of $\frac{\text{Actual Error}}{\text{Probable Error}}$ put a series of distinctive symbols, as (1), (2) &c., or any other convenient form; and on the left side, omit the "Discordance." Then combine and solve the equations precisely as for numerical determination of δ.*Venus*' RA, δ.*Venus*' NPD, &c., always using the numerical values on the left side, and the symbols (1), (2) &c., with proper numerical factors, on the right side. The solution will give

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$$\delta.\textit{Venus' } \mathrm{RA} = A_1 \times (1) + A_2 \times (2) + \&c.$$
$$\delta.\textit{Venus' } \mathrm{NPD} = B_1 \times (1) + B_2 \times (2) + \&c.$$
$$\&c. \quad = \quad \&c.$$

where A_1, A_2 &c., B_1, B_2 &c., are numerical quantities. Then, as the probable error of each of the quantities (1), (2) &c., is = 1, we have

$$\text{Probable error of } \delta.\textit{Venus' } \mathrm{RA} = \sqrt{(A_1)^2 + (A_2)^2 + \&c.},$$

and so for the others.

The equations are solved by precisely the same process which has given the values of δ. *Venus'* RA &c.; and the factors (1), (2) &c., are abstract numbers. Hence, it is easily seen that the unit in the expressions for Probable Errors of δ.*Venus'* RA &c. is the same as the unit in the expressions for δ.*Venus'* RA &c., that is, as is most likely, $1''\cdot 00$.

27. When the equations are solved (whether the entire collection, or separate groups as suggested in Article 24), the inferred values of δ.♀RA, δ.♀NPD, δ.R, δ.*r*, δ.P, ought to be substituted in the equations of Articles 16 and 22, and the residual errors ought to be exhibited. The study of these residual errors will be peculiarly the work of the Superintending Astronomer, who shall be charged with the task of communicating to the world the important inferences deducible from the Transit of *Venus* 1874. The first of these, it is almost unnecessary to observe, is the value of Solar Parallax. The concomitant inferences, as to the corrections of the Tables of the Earth and *Venus*, will interest only the professional gravitational astronomer. But there will be other points, not of a numerical nature, but which are extracted most surely from numerical statements, to be ascertained from these comparisons:—The effect of the magnitude and excellence of the telescope for eye-views; the value and arrangement of the photographic system and pictures; the principles for the selection of stations; and other points which will present themselves to that scientific authority. The examination of these, it is to be hoped, would tend materially to an advance of accuracy in the results of the Transit of *Venus* 1882.

Finally, I would suggest for consideration the following points:—

(I). It is desirable that the Observations made in each of the National Expeditions should be printed with the least possible delay, under the superintendence of the head of the Expedition; and should be at once distributed to the principal scientific institutions and libraries, and to the persons officially interested in the Transit.

(II). It is desirable that these Observations should be printed in Quarto, of such size that all could be conveniently bound together. There is little difference in the sizes of the French

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Memoirs, the Berlin Memoirs, the Smithsonian Memoirs, the Royal Astronomical Society's Memoirs; the size of the *Philosophical Transactions* is somewhat larger.

(III). A brief history of each Expedition should be given, with the names and offices of the persons employed; and description of the localities, accompanied by maps if necessary, and by such statements as may lead to identification of the localities of observations.

(IV). In the accounts of telegraphic operations, it may not be necessary to give details, if the instruments and methods are described, and a few observations are given sufficient for check of the fundamental conclusions; in the accounts of Transits, it is sufficient to give description of instruments and methods, constants of adjustments, and tables of clock-errors; and analogous abstracts of comparisons of chronometers, &c.

(V). But it is necessary to give in the fullest detail everything that bears upon the actual observation of contacts, or upon the observer's impression at the time of making the observation, or upon the micrometer-measures, or upon the photographs and the measures of the photographs, &c.; with sufficient description of the instruments and their adjustments at the time. Clock-time and Local Sidereal Time are to be given for every observation.

(VI). Reference must be made to the place of deposit of the original documents, observations, calculations, and instruments, of each National Expedition.

(VII). A large portion of the calculations sketched in the preceding Articles can be best made under the superintendence of the head of each National Expedition, and these should be printed and distributed as part of the Observations mentioned above.

(VIII). It is to be hoped that some astronomer of eminence may undertake to superintend, in some degree, the publications which I have suggested; and to make the final combination of observations and decide on the ultimate result. I can without difficulty fix on the foreign astronomer in whose hands I should be glad to see this work placed.

Royal Observatory, Greenwich,
1875, *Jan.* 19.